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Meson Spectrum

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Using the Klein–Gordon equation with a box potential, a mass formula describing the family of nonflavored meson states with $I = 1$ is derived. The energy levels calculated agree with those observed within an accuracy of ~5%. In the model discussed quarks behave like tachyons.

1. INTRODUCTION

In papers by Kang and Schnitzer (1975) and Ram and Halasa (1979), meson spectra have been calculated using the potentials $V(r) = ar + b$ or $V(r) = ar^2 + b$, respectively, as the fourth component of a four-vector in the Klein–Gordon equation for the energy eigenstates ($c = \hbar = 1$)

$$[\nabla^2 + \tfrac{1}{4}(E - V)^2 - m^2]\phi(\mathbf{r}) = 0$$

$$E - V = 2(p^2 + m^2)^{1/2} \tag{1.1}$$

where m is the rest mass of the quark or antiquark. The motivation for using the linear or harmonic oscillator potential comes from the field-theoretic arguments or phenomenological viewpoint. Following this approach, we assume the box potential

$$V(r) = 0, \qquad 0 \leq r < R$$
$$V(r) = \infty, \qquad R \leq r$$

The box potential represents a certain approximation of the confining potential and the motivation for this choice comes from the feature of the potentials used by Kang and Schnitzer (1975) and Ram and Halasa (1979), where $V(r \to \infty) \to \infty$, and from a good agreement of the mass formula obtained with the meson states observed. In this approach, of course, the

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retardation and spin-dependent correction are neglected (Kang and Schnitzer, 1975). Putting

$$\phi(\mathbf{r}) = f(r) Y_l^m(\vartheta, \varphi)$$

we find that the radial wave function in our case satisfies the equation

$$\left[r^2 \frac{d^2}{dr^2} + 2r \frac{d}{dr} - l(l+1) + p^2 r^2 \right] f(r) = 0 \tag{1.2}$$

with $p^2 = \frac{1}{4}E^2 - m^2$, under the condition $f(r = R) = 0$. The solution obtained reads (Kamke, 1951)

$$f_{ln}(r) = Cr^{-1/2} J_{l+1/2}(\alpha_{ln} r/R), \qquad p_{ln}^2 = \alpha_{ln}^2 R^{-2} \tag{1.3}$$

where α_{ln} are the roots of Bessel functions $J_{l+1/2}$. According to (1.2) and (1.3), we have

$$M_n^l = (\alpha_{ln}^2 K^2 + 4m^2)^{1/2} \tag{1.4}$$

with $M_n^l = E_n^l$, $K = 2/R$.

2. NONFLAVORED MESON STATES $I = 1$

Assuming the quark masses $m_d = m_u$, we can use the formula (1.4) to calculate the excited states of the bound states $q\bar{q}$ for nonflavored mesons with $I = 1$. We may fix the two free parameters m and K by choosing as input the masses of two different meson states. Inspecting the family of π mesons, we find only two well-established states with l defined uniquely, namely $\pi(137)$, $l = 0$, and $\pi_2(1680)$, $l = 2$. The $\pi(1300)$ is not a well-established state (Particle Data Group, 1986). Therefore we take as input

$$M_1^0 = M_\pi = 137.3 \text{ MeV}, \qquad M_5^2 = M_{\pi_2} = 1680 \text{ MeV} \tag{2.1}$$

Then, according to (1.4),

$$M_n^l = [M_\pi^2 + (\alpha_{ln}^2 - \alpha_{01}^2) K^2]^{1/2} \tag{2.2}$$

where $K^2 = 8259 \text{ MeV}^2$ and for the radius of the box we obtain $R = 4.5 \times 10^{-13}$ cm.

In the state (ln) the momentum squared of the particle in the box is given by (1.3) and the angular momentum squared by $L_l^2 = l(l+1)$. However, the quantities p^2 and L^2 are not independent. Therefore, in a physical state observed the quantum numbers l and n are not independent. As an example, consider the hydrogen atom. For all physical states of the hydrogen atom

$$l < n \tag{2.3}$$

We see that this condition is also satisfied for our inputs (2.1). Now we will show that the mass formula (2.2) together with the condition (2.3) describes well the family of meson states with $I = 1$.

In Table I the energy levels M_n^l calculated according to (2.2) and (2.3) are compared with those observed (Particle Data Group, 1986). For the states $J^P = 0^-$, 0^+, 1^+, and 2^- the value of l is defined uniquely (underlined states). For all meson states located in Table I the correlation coefficient

$$r_{xy} = \frac{\Sigma(x_i - \bar{x})\Sigma(y_i - \bar{y})}{[\Sigma(x_i - \bar{x})^2 \Sigma(y_i - \bar{y})^2]^{1/2}} = 0.945$$

and the regression coefficient

$$b_{y/x} = r_{xy}\left[\frac{\Sigma(y_i - \bar{y})^2}{\Sigma(x_i - \bar{x})^2}\right]^{1/2} = 0.937$$

have been calculated. It is seen that the statistical relevance of the agreement is high. On the other hand, the model in question reveals some unexpected features. According to (1.4) and (2.2),

$$4m^2 = M_\pi^2 - \alpha_{01}^2 K^2$$

and for the value of K found we have $4m^2 = -62{,}683\ \mathrm{MeV}^2$. Hence, in the model discussed quarks behave like tachyons. A simple box model used to calculate possible gravitational perturbations of mesons leads to an analogous result: $\alpha = M^2c^2 < 0$ (Burcev, 1985).

Table I. Calculated and Observed Energy Levels M_n^l of Nonflavored Meson States with $I = 1$[a]

	M_n^l (MeV)							
l	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	J^{PC}	$^{2s+1}l_J$
0	137	513	819	1,115	1,405	1,695		
	$\pi(\underline{137})$				$\pi(\underline{1{,}300})$		0^{-+}	$^1S_0$
			$\rho(770)$			$\rho(1{,}590)$	1^{--}	$^3S_3$
1		656	958	1,254	1,545	1,834		
			$a_0(\underline{983})$				0^{++}	$^3P_0$
				$b_1(\underline{1{,}233})$			1^{+-}	$^1P_1$
				$a_1(\underline{1{,}275})$			1^{++}	$^3P_1$
				$a_2(1{,}318)$			2^{++}	$^3P_2$
2			1,011	1,387	1,680	1,970		
							1^{--}	$^3D_1$
					$\pi_2(\underline{1{,}680})$		2^{-+}	$^1D_2$
							2^{--}	$^3D_2$
					$\rho_3(1{,}691)$		3^{--}	$^3D_3$
3				1,517	1,811	2,103		
							2^{++}	$^3F_2$
							3^{+-}	$^1F_3$
							3^{++}	$^3F_3$
							4^{++}	$^3F_4$

[a]Meson states with l defined uniquely are underlined.

3. CONCLUSIONS

It is well known that there exists the possibility for a description of an elementary particle that has constituents if those constituents behave like tachyons [see Recami (1986) and references therein]. Assuming that quarks behave like tachyons, one should replace equations (1.1) as follows:

$$[\nabla^2 + \tfrac{1}{4}(E - V)^2 + m^2]\phi(\mathbf{r}) = 0$$
$$E - V = 2(p^2 - m^2)^{1/2} \tag{3.1}$$

where m is the real rest mass of a quark-tachyon. Then the mass formula (1.4) takes the form

$$M_n^l = (\alpha_{ln}^2 K^2 - M^2)^{1/2} \tag{3.2}$$

with $M = 2m$. Now, from (2.2) and (3.2) for the rest masses of the presumed quark-tachyons we obtain

$$m_d = m_u = \tfrac{1}{2}(\alpha_{01}^2 K^2 - M_\pi^2)^{1/2} = 125 \text{ MeV}$$

The good agreement of the energy levels calculated with those observed (Table I) seems not to be purely coincidental. In the model discussed mesons appear as excited states of the system "box with quark-tachyons." For the meson states with $I = 0$ and $I = \frac{1}{2}$ the simple equations (3.1) cannot be used because evidently $m_s \neq m_d$, m_u. Despite the fact that the model represents a crude approximation only, the discussion of the results obtained on a broader level and in more detail might be useful.

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